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THE GLOBAL DIFFUSION OF IDEAS

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We provide a tractable, quantitatively-oriented theory of innovation and technology diffusion to explore the role of international trade in the process of development. We model innovation and diffusion as a process involving the combination of new ideas with insights from other industries or countries. We provide conditions under which each country's equilibrium frontier of knowledge converges to a Fréchet distribution, and derive a system of differential equations describing the evolution of the scale parameters of these distributions, that is, countries' stocks of knowledge. The model remains tractable with many asymmetric countries and generates a rich set of predictions about how the level and composition of trade affect countries' frontiers of knowledge. We use the framework to quantify the contribution of bilateral trade costs to long-run changes in TFP and individual post-war growth miracles. For our preferred calibration, we find that both gains from trade and the fraction of variation of TFP growth accounted for by changes in trade more than double relative to a model without diffusion.

KEYWORDS: Economic growth, diffusion, trade, knowledge.

ECONOMIC MIRACLES are characterized by protracted growth of per-capita income and productivity as well as increases in trade flows. The experiences of South Korea in the postwar period and the recent performance of China are prominent examples. These experiences suggest an important role played by openness in the process of development.[1] Yet quantitative trade models relying on standard static mechanisms imply relatively small gains from openness and, therefore, cannot account for growth miracles.[2] These findings call for alternative channels through which openness can affect development. In this paper, we present and analyze a tractable, quantitatively-oriented model of an alternative

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[1] Sachs and Warner (1995), Dollar (1992), Ben-David (1993), Coe and Helpman (1995), and Frankel and Romer (1999) suggested a strong relationship between openness and growth, although Rodriguez and Rodrik (2001) subsequently argued that many estimates in the literature suffered from econometric issues including omitted variables, endogeneity, and lack of robustness. More recent contributions to the literature have developed strategies to overcome some of these issues. To estimate the impact of trade on growth, Feyrer (2009a,b) studied the natural experiments of the decade-long closing of the Suez Canal in the 1970s and the long run decline in the cost of shipping goods by air, each of which had larger impacts on some pairs of countries than others, and Pascali (2017) studied the introduction of the steamship which affected some trade routes more than others. See also Lucas (2009b) and Wacziarg and Welch (2008), and Donaldson (2015) for reviews of the literature.

[2] See Connolly and Yi (2015) for a quantification of the role of trade on Korean's growth miracle. Atkeson and Burstein (2010) also found relatively small effects in a model with innovation.

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mechanism: the impact of openness on the creation and diffusion of best practices across countries.[3]

We model innovation and diffusion as a process involving the combination of new ideas with insights from other industries and countries. Insights occur randomly and result from local interactions among producers. In our theory, openness affects the creation and diffusion of ideas by determining the distribution from which producers draw their insights. Our theory is flexible enough to incorporate and contrast different channels through which ideas may diffuse across countries. We focus on two main channels: (i) insights are drawn from those that sell goods to a country, (ii) insights are drawn from technologies used domestically. In our model, openness to trade affects the quality of the insights drawn by producers because it determines the set of sellers to a country and the set of technologies used domestically.

In this context, we provide conditions under which the distribution of productivity among producers within each country always converges to a Fréchet distribution, no matter how trade barriers shape individual producers' local interactions. As a consequence, the state of knowledge within a country can be summarized by the level of this distribution, which we call the country's stock of knowledge. The model is thus compatible with the Eaton and Kortum (2002) machinery which has been useful in studying trade flows in an environment with many asymmetric countries. We show that the change in a country's stock of knowledge can be characterized in terms of only its trade shares, its trading partners' stocks of knowledge, and parameters. This both yields qualitative insights and enables us to use actual trade flows to discipline the role of trade and geography in shaping idea flows and growth.

Starting from autarky, opening to trade results in a higher temporary growth rate, and permanently higher level, of the stock of knowledge, as producers are exposed to more productive ideas. We separate the gains from trade into static and dynamic components. The static component consists of the gains from increased specialization and comparative advantage, whereas the dynamic component consists of the gains that operate through the flow of ideas.

We first explore an environment in which producers in a country gain insights from those that sell goods to the country, following Alvarez, Buera, and Lucas (2013). With this specification of learning, the dynamic gains from reducing trade barriers are qualitatively different from the static gains. The dynamic gains are largest for countries that are relatively closed, whereas the static gains are largest for countries that are already relatively open. For a country with high trade barriers, the marginal import tends to be made by a foreign producer with high productivity. While the high trade costs imply that the static gains from trade remain relatively small, the insights drawn from these marginal producers tend to be of high quality. In contrast, for a country close to free trade, the reduction in trade costs leads to large infra-marginal static gains from trade, but the insights drawn from the marginal producers are likely to have lower productivity and generate lower quality ideas.

At two extremes, our model nests a simple version of the Kortum (1997) model of pure innovation and one closely related to the Alvarez, Buera, and Lucas (2008, 2013) model

[3]Parente and Prescott (1994) and Klenow and Rodriguez-Clare (2005) argued that without some form of international spillovers or externalities, growth models have difficulty accounting for several facts about growth and development. Each argue that these facts can be explained by catchup growth to a world frontier of knowledge, an idea that goes back to at least Nelson and Phelps (1966). Comin and Hobijn (2010) documented large cross-country differences in the speed with which frontier technologies are adopted and Comin, Dmitriev, and Rossi-Hansberg (2012) showed that the speed of diffusion declines with distance.

of pure diffusion. We span these two extremes by varying a single parameter, β, which we label the strength of diffusion. The parameter β measures the contribution of insights from others to the productivity of new ideas. One striking observation is that, for either of these two extremes, if a moderately open country lowers its trade costs, the resulting dynamic gains from trade are relatively small, whereas when β is in an intermediate range, the dynamic gains are larger. When β is small, insights from others are relatively unimportant; there are strong diminishing returns in how the quality of an insight contributes to the productivity of a new idea. It follows immediately that dynamic gains tend to be small. When β is larger, insights from others are more central. We show that the gains relative to autarky approach infinity as β approaches 1. A more subtle implication is that in the limiting model as β approaches the extreme of one, catchup growth is concentrated near autarky. In particular, we show that for a country not in autarky, the *marginal* catchup growth from a reduction in trade costs approaches zero as β approaches one.[4] In this limit, a moderately open country is much better off than it would be in autarky, but further reductions in trade costs have little impact; the dynamic gains are dominated by insights from the most productive foreign firms, who would export as long as trade costs are finite. As a consequence, it is only when β is in an intermediate range that the dynamic gains from trade are both sizable and would result from reductions in trade costs in the empirically relevant range.

We contrast this with a second channel, that individuals may draw insights from others that produce domestically, following Sampson (2016) and Perla, Tonetti, and Waugh (2015). In this setting, lower trade barriers increase domestic competition and improve the distribution of productivity among those that continue to produce domestically, raising the quality of insights manager might draw from. We show that the two channels are quite different. Under this specification of learning, the long-run dynamic gains from trade simply amplify the static gains, more so when β is larger so that insights from others contribute more to the productivity of new ideas.

To explore the ability of the theory to account for the evolution of the world distribution of productivity, we specify a quantitative version of the model that includes nontraded goods, intermediate inputs, and labor equipped with capital and education, and use it to study the ability of the theory to account for the evolution of TFP between 1962 and 2000. Following Waugh (2010), we use panel data on trade flows and relative prices to calibrate the evolution of bilateral trade costs, and take the evolution of population, physical and human capital, that is, equipped labor, from the data. Given the evolution of trade costs and equipped labor, our model predicts the evolution of each country's TFP.

The predicted relationship between trade and TFP depends on the value of β, the strength of diffusion, which indexes the contribution of insights drawn from others to the productivity of new ideas. While we provide a simple heroic strategy to calibrate this parameter, our main approach is to simulate the model for various alternative values and explore how well the model can quantitatively account for cross-country income differences and the evolution of countries' productivity over time.

In line with the theoretical results, the role of trade in accounting for the dispersion of TFP growth is highest for intermediate values of the diffusion parameter, β. There are

[4]In the environment studied by Alvarez, Buera, and Lucas (2013), both the steady state growth rate and the mass in the right tail of countries' productivity distributions are proportional to the number of countries not in autarky; trade costs have no other impact on these objects. Note that in their model the distribution of productivity is not Fréchet outside the cases of the autarky and costless trade with symmetric countries, so that trade affects features of the distribution beyond the right tails.

several ways one might measure the contribution of changes in trade barriers to changes in TFP. When insights are drawn from sellers, we find that, across measures, the contribution of trade is up to three times as large when the model allows for dynamic gains from trade. For our preferred calibration, we find that both the gains from trade and the fraction of variation of TFP growth accounted for by changes in trade more than double relative to a model without diffusion. The quantitative model is particularly capable of explaining a substantial part of the evolution of TFP in growth miracles, accounting for over a third of the TFP growth in China, South Korea, and Taiwan.

Literature Review. Our work builds on a large literature modeling innovation and diffusion of technologies as stochastic processes, starting from the earlier work of Jovanovic and Rob (1989), Jovanovic and MacDonald (1994), Kortum (1997), and recent contributions by Alvarez, Buera, and Lucas (2008), Lucas (2009a), and Luttmer (2007, 2012).[5] We are particularly related to recent applications of these frameworks that study the connection between trade and the diffusion of ideas (Lucas (2009b), Alvarez, Buera, and Lucas (2013), Perla, Tonetti, and Waugh (2015), Sampson (2016)).

In our model, the productivity of new ideas combines both insights from others and an original component.[6] As discussed earlier, our theory captures the models in Kortum (1997) and Alvarez, Buera, and Lucas (2008, 2013) as special, and we argue, quantitatively less promising cases. In Kortum (1997), there is no diffusion of ideas, and thus no dynamic gains from trade. In Alvarez, Buera, and Lucas (2013), when trade barriers are finite, changes in trade barriers have no impact on the tail of the distribution of productivity and, therefore, the model has more limited success in providing a quantitative theory of the level and transitional dynamics of productivity. In addition, for the intermediate cases that are the focus of our analysis, $\beta \in [0, 1)$, the frontier of knowledge converges to a Fréchet distribution.[7] This allows us use the machinery of Eaton and Kortum (2002), enabling us to quantify the role of both trade barriers and geography in the flow of ideas.

Eaton and Kortum (1999) also built a model of the diffusion of ideas across countries in which the distribution of productivities in each country is Fréchet, and where the evolution of the scale parameter of the Fréchet distribution in each country is governed by a system of differential equations. In their work, insights are drawn from the distribution of potential producers in each country, according to exogenous diffusion rates which

[5] Lucas and Moll (2014) and Perla and Tonetti (2014) extended these models by endogenizing search effort. The main text abstracts from search effort, but Online Appendix D (Buera and Oberfield (2020)) studies how trade barriers affect incentives to innovate. Following Bernard, Eaton, Jensen, and Kortum (2003) we focus on a decentralization in which producers engage in Bertrand competition, and each producer earns profit on sales to any destination to which that producer is the lowest-cost provider of a good. Motivated by the potential for profit, producers hire labor to generate new ideas. In this environment, we extend the result of Eaton and Kortum (2001) that on any balanced growth path, each country's research effort is independent of trade barriers. Chiu, Meh, and Wright (2017) studied information issues in the transfer of ideas, a dimension that we abstract from.

[6] See König, Lorenz, and Zilibotti (2016) and Benhabib, Perla, and Tonetti (2017) for models in which individuals can choose either to imitate or to innovate.

[7] Alvarez, Buera, and Lucas (2013) studied a model with $\beta = 1$. In their model, the limiting distribution of productivities is only Fréchet in the extreme cases of autarky and costless trade among symmetric countries. In our model, the limiting distribution is Fréchet for any $\beta \in [0, 1)$ and any configuration of trade costs. This makes our framework significantly more tractable and amenable to quantitative exploration with many heterogeneous countries. At the same time, the departure from Fréchet in Alvarez, Buera, and Lucas (2013) implies that features of the distribution beyond the right tails matter for the gains from trade, and dynamic gains from trade could be sizable if the elasticity of substitution across goods is low.

are estimated to be country-pair specific, although countries are assumed to be in autarky otherwise. Therefore, changes in trade do not affect the diffusion of ideas. In such a model, trade and diffusion are substitutes. Here, however, diffusion only happens because of trade—those are the local interactions through which producers gain knowledge. This leads to different predictions about the way trade patterns covary with diffusion patterns.

In analyzing examples where learning is solely from domestic producers, our paper relates to the recent work by Perla, Tonetti, and Waugh (2015) and Sampson (2016).[8] They consider monopolistically-competitive trade models with symmetric countries, firm entry, exit, and technology adoption. Entrants and inactive firms learn from the set of domestic producers. In their models, trade barriers affect the *growth rate* on a balanced growth path only if producers are required to pay a fixed cost to export, that is, if there is selection into exporting. Instead, we consider a Ricardian trade model which features selection into producing domestically even in the absence of fixed exporting costs, where trade barriers affect the *level* of income on a balanced growth path. Further, their analysis emphasizes the intensity of adoption, entry, and exit, whereas ours emphasizes the composition of insights.

Our works relates to a large literature studying the connection between trade and growth, including the early contributions by Grossman and Helpman (1991) and Rivera-Batiz and Romer (1991). Closest to ours is Grossman and Helpman (1991), who consider a small open economy in which technology is transferred from the rest of the world as an external effect, and the pace of technology transfer is assumed to depend on the volume of trade. Our model incorporates this channel along with several others and embeds the mechanism in a quantitative framework. In addition, our paper relates to a large empirical literature providing evidence on the relationship between openness and diffusion of technologies. The law of motion of the stock of knowledge in our model is consistent with the early evidence discussed in Coe and Helpman (1995) and Coe, Helpman, and Hoffmaister (1997) about the importance of knowledge spillovers through trade. See Keller (2009) for a recent review of this empirical literature, considering alternative channels, including trade and FDI. Our findings are also related to Baldwin and Robert-Nicoud (2008) who study a wide class of models relating trade to incentives to innovate and find that small differences in assumptions about spillovers can lead to different implications for the impact of a trade liberalization on growth.

The model shares some features with Oberfield (2018), which models the formation of supply chains and the economy's input-output architecture. In that model, entrepreneurs discover methods of producing their goods using other entrepreneurs' goods as inputs.[9]

1. IDEA DIFFUSION WITH A GENERAL SOURCE DISTRIBUTION

We begin with a description of technology diffusion in a single country given a general source distribution. The source distribution describes the set of insights that producers might access. In the specific examples that we explore later in the paper, the source distribution will depend on the profiles of productivity across all countries in the world, but in this section we assume only that it satisfies weak tail properties. Given these assumptions, we show that the equilibrium distribution of productivity within an economy converges to

[8] Sampson (2016) also included an extension that incorporates learning from sellers.

[9] Here, the evolution of the distribution of marginal costs depends on a differential equation summarizing the history of insights that were drawn. In Oberfield (2018), the distribution of marginal costs is the solution to a fixed-point problem, as each producer's marginal cost depends on her potential suppliers' marginal costs.

a Fréchet distribution, and derive a differential equation describing the evolution of the scale parameter of this distribution.

We consider an economy with a continuum of goods $s \in [0, 1]$. For each good, there is large set of potential producers with different technologies to produce the good. We will later study an environment in which the producers engage in Bertrand competition, so that (barring ties) at most one of these producers will actively produce. A producer is characterized by the productivity of her idea. An idea to produce good s with productivity q is a labor-only, linear production technology

$$y(s) = ql(s), \tag{1}$$

where $l(s)$ is the labor input and $y(s)$ is output of good s.

We now describe the dynamics of knowledge. We model diffusion as a process involving the random interaction among producers of different goods or countries. New ideas arrive to potential producers of each good stochastically and exogenously.[10] Each idea is a technology to produce a particular good with productivity q. New ideas build on insights from others in the economy, but there is randomness in the adaptation of that insight. More formally, when a new idea arrives, the productivity of the idea is $q = zq'^{\beta}$, which has two random components. There is an insight drawn from another producer, q', which is drawn from the source distribution $G_t(q')$. There is also an original component, z, drawn from an exogenous distribution. We assume that the arrival rate of ideas with original component greater than z is $A_t(z)$.[11]

This process captures the fact that interactions with more productive individuals tend to lead to more useful insights, but it also allows for randomness in the adaptation of others' techniques to alternative uses. The latter is captured by the random variable z. An alternative interpretation of the model is that z represents an innovator's "original" random idea, which is combined with random insights obtained from other technologies. The parameter β captures the contribution of the quality of insights from others to the productivity of new ideas.[12] For concreteness, consider the average productivity among ideas inspired by a producer with productivity q_1 relative to the average productivity among ideas inspired by a producer with q_2, with $q_1 > q_2$. This ratio is $(q_1/q_2)^{\beta} > 1$. If β is larger, the ratio is larger, so the relative quality of the insight is a more important determinant of the productivity of a new idea. A second role played by β is that it indexes the ease of improving on higher quality insights.[13] While better insights tend to generate better ideas, if $\beta < 1$, the ratio of the productivity of the new idea to the productivity of the insight, $z(q')^{\beta}/q'$, is decreasing in the quality of the insight, q'. The smaller is β, the more sharply the ratio declines with q'.

[10] In the Online Appendix D, we endogenize the arrival of new ideas as resulting from research.

[11] From the perspective of this section, both the insight, q', and the original contribution, z are drawn from exogenous distributions. The distinction between these distributions will become clear once we consider specific examples of source distributions, in which case the source distribution will be an *endogenous* function of countries' frontiers of knowledge.

[12] If $\beta = 0$, our framework simplifies to a version of the model in Kortum (1997) with exogenous search intensity. The framework also nests the model of diffusion in Alvarez, Buera, and Lucas (2008) with stochastic arrival of ideas if $\beta = 1$, A_t is degenerate, and $G_t = F_t$.

[13] Jones (1995), and more recently, Atkeson and Burstein (2011) featured analogous parameters that index the degree of intertemporal spillovers in innovation. Because those models do not model the spillovers explicitly, the composition of insights is irrelevant and the parameter plays only the second role. Here, the composition of insights is central.

The economy's productivity depends on the frontier of knowledge. The frontier of knowledge is characterized by the function $F_t(q)$ which denotes the fraction of goods for which no producer's productivity exceeds q. Given the frontier of knowledge at time t, $F_t(q)$, the source distribution, $G_t(q')$, and the exogenous arrival rates of ideas, $A_t(z)$, the frontier of knowledge at time $t+\Delta$ satisfies

$$1 - F_{t+\Delta}(q) = \left[1 - F_t(q)\right] + F_t(q) \int_t^{t+\Delta} \int A_\tau\left(\frac{q}{q'^{\beta}}\right) dG_\tau(q')\, d\tau.$$

In words, the fraction of goods for which the frontier productivity exceeds q at $t+\Delta$ is given by those for which the frontier exceeds q at t, $1 - F_t(q)$, and, among the remainder, those for which a new idea with productivity exceeding q arrives between t and $t+\Delta$. To find the arrival rate of such ideas, note that given any insight q', the arrival rate of ideas that, in combination with that insight, would deliver productivity $zq'^{\beta} > q$ is $A(q/q'^{\beta})$. Integrating over possible insights gives $\int A(q/q'^{\beta})\, dG_\tau(q')$, the arrival rate of ideas with productivity greater than q.

Rearranging and taking the limit as $\Delta \to 0$, we obtain an expression characterizing the evolution of the frontier of knowledge:

$$\frac{d}{dt} \ln F_t(q) = \lim_{\Delta \to 0} \frac{F_{t+\Delta}(q) - F_t(q)}{\Delta F_t(q)} = -\int_0^{\infty} A_t\left(q/q'^{\beta}\right) dG_t\left(q'\right). \tag{2}$$

That is, the fraction of goods for which the frontier productivity is weakly less than q declines with the arrival of ideas with productivity greater than q.

To gain tractability, we assume that the arrival rate of ideas with original component greater than z follows a power law.

ASSUMPTION 1: *The arrival rate of ideas with original component greater than z is*

$$A_t(z) = \alpha_t z^{-\theta}.$$

Proposition 1 describes the evolution of the frontier of knowledge and shows that for any initial distribution, the appropriately-scaled frontier of knowledge converges asymptotically to a Fréchet distribution with shape parameter θ.[14] Proposition 1 imposes the additional restriction that the source distribution G_t has a sufficiently thin tail. Later when we endogenize the source distribution, Assumption 3 will be sufficient to guarantee that this restriction is satisfied for the examples we consider.

PROPOSITION 1: *Suppose Assumption 1 holds and that at each t, $\lim_{q\to\infty} q^{\beta\theta}[1 - G_t(q)] = 0$. Then the frontier of knowledge evolves as*

$$\frac{d \ln F_t(q)}{dt} = -\alpha_t q^{-\theta} \int_0^{\infty} x^{\beta\theta}\, dG_t(x). \tag{3}$$

Define $\lambda_t \equiv \int_{-\infty}^{t} \alpha_\tau \int_0^{\infty} x^{\beta\theta}\, dG_\tau(x)\, d\tau$. If $\lim_{t\to\infty} \lambda_t = \infty$, then

$$\lim_{t\to\infty} F_t\left(\lambda_t^{1/\theta} q\right) = e^{-q^{-\theta}}.$$

[14]In Buera and Oberfield (2016), we showed that analogous results could be derived if Assumption 1 were replaced by the weaker assumption that the right tail of A_t was regularly varying, $\lim_{z\to\infty} \frac{A_t(z)}{z^{-\theta}} = \alpha_t$ and the arrival rate of new ideas grew arbitrarily large.

PROOF: Equation (3) follows directly from (2) after imposing Assumption 1. Solving the differential equation (3) gives $F_t(q) = F_0(q)e^{-(\lambda_t-\lambda_0)q^{-\theta}}$. Evaluating this at $\lambda_t^{1/\theta}q$ gives $F_t(\lambda_t^{1/\theta}q) = F_0(\lambda_t^{1/\theta}q)e^{-(\lambda_t-\lambda_0)\lambda_t^{-1}q^{-\theta}}$. This implies that, asymptotically, $\lim_{t\to\infty} F_t(\lambda_t^{1/\theta}q) = e^{-q^{-\theta}}$. *Q.E.D.*

Motivated by this proposition, we make the additional assumption that the initial frontier of knowledge follows a Fréchet distribution.

ASSUMPTION 2: *The initial frontier of knowledge is* $F_0(q) = e^{-\lambda_0 q^{-\theta}}$.

This assumption and Proposition 1 imply that the frontier of knowledge retains its shape, taking the form $F_t(q) = e^{-\lambda_t q^{-\theta}}$, and that the scale parameter evolves according to

$$\dot{\lambda}_t = \alpha_t \int_0^\infty x^{\beta\theta}\, dG_t(x). \tag{4}$$

For reasons that will be explained shortly, we call λ_t the stock of knowledge. Equation (4) says that the stock of knowledge increases faster when ideas arrive more quickly (α_t) and when insights are drawn from a better source distribution. The latter is manifested in the fact that λ increases faster when the $\beta\theta$ moment of the source distribution is larger. When β is larger, insights make a greater contribution to the productivity of new ideas and, in particular, the best insights play a more important role.

In the rest of the paper, we analyze alternative models for the source distribution G_t. A simple example that illustrates basic features of more general cases is $G_t(q) = F_t(q)$. This corresponds to the case in which diffusion opportunities are randomly drawn from the set of domestic best practices across all goods. In a closed economy, this set equals the set of domestic producers and sellers. We impose one further assumption which, in the economies we study, is necessary to keep $\dot{\lambda}$ finite.

ASSUMPTION 3: $\beta \in [0, 1)$.

In our example with $G_t(q) = F_t(q)$, (4) becomes

$$\dot{\lambda}_t = \alpha_t \Gamma(1-\beta)\lambda_t^{\beta},$$

where $\Gamma(u) \equiv \int_0^\infty x^{u-1}e^{-x}\, dx$ is the Gamma function. Here, the contribution of insights from others to growth in the stock of knowledge is summarized by the term $\Gamma(1-\beta)\lambda_t^{\beta}$, where λ_t is the stock of knowledge among those from whom one learns. Again, when β is larger, it is relatively more important that insights are drawn from a better source distribution.

Growth in the long-run is obtained in this framework if the arrival rate of insight grows over time, $\alpha_t = \alpha_0 e^{\gamma t}$. In this case, the scale of the Fréchet distribution λ_t grows asymptotically at rate $\frac{\gamma}{1-\beta}$, and per-capita GDP grows at the rate $\frac{\gamma}{(1-\beta)\theta}$. When $\beta < 1$, it is more difficult to improve on a better quality insight. As the frontier of knowledge improves, ideas must arrive more quickly in order to maintain a constant rate of growth. This feature—shared with the semiendogenous growth models of Jones (1995), Kortum (1997), and Atkeson and Burstein (2011)—implies that the economy's growth rate depends on the growth of the arrival of ideas and that an increase in the arrival rate of ideas leads to level effects rather than growth effects. The evolution of the detrended stock of knowledge $\hat{\lambda}_t = \lambda_t e^{-\gamma/(1-\beta)t}$ can be summarized in terms of the detrended arrival of ideas $\hat{\alpha}_t = \alpha_t e^{-\gamma t}$,

$$\dot{\hat{\lambda}}_t = \hat{\alpha}_t \Gamma(1-\beta)\hat{\lambda}_t^{\beta} - \frac{\gamma}{1-\beta}\hat{\lambda}_t,$$

and on a balanced growth path on which $\hat{\alpha}$ is constant, the detrended stock of knowledge is

$$\hat{\lambda} = \left[\frac{\hat{\alpha}(1-\beta)}{\gamma}\Gamma(1-\beta)\right]^{\frac{1}{1-\beta}}.$$

2. INTERNATIONAL TRADE

Consider a world in which n economies interact through trade and ideas diffuse through contact with other producers. Given the results from the previous section, the static trade theory is given by the standard Ricardian model in Eaton and Kortum (2002), Bernard et al. (2003), and Alvarez and Lucas (2007), which we briefly introduce before deriving the equations which characterize the evolution of countries' knowledge in the world economy.

In each country, consumers have identical preferences over a continuum of goods. We use $c_i(s)$ to denote the consumption of a representative household in i of good $s \in [0, 1]$. Utility is given by $u(C_i)$, where the the consumption aggregate is

$$C_i = \left[\int_0^1 c_i(s)^{\frac{\varepsilon-1}{\varepsilon}}\,ds\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

so goods enter symmetrically and exchangeably. We assume that $\varepsilon - 1 < \theta$, which guarantees the price level is finite. Let $p_i(s)$ be the price of good s in i, so that i's ideal price index is $P_i = [\int_0^1 p_i(s)^{1-\varepsilon}\,ds]^{\frac{1}{1-\varepsilon}}$.

In each country, individual goods can be manufactured by many producers, each using a labor-only, linear technology (1). As discussed in the previous section, provided that the evolution of knowledge in each country satisfies Assumptions 1–3, the frontier of knowledge in each country at any subsequent date is described by a Fréchet distribution with curvature θ and a country-specific scale λ_i, $F_i(q) = e^{-\lambda_i q^{-\theta}}$. Transportation costs are given by the standard "iceberg" assumption, where κ_{ij} denotes the units that must be shipped from country j to deliver a unit of a good to country i, with $\kappa_{ii} = 1$ and $\kappa_{ij} \geq 1$. Let w_i denote the wage in country i. For a producer with productivity q in country j, the cost of providing one unit of the good to country i is $\frac{w_j \kappa_{ij}}{q}$. Producers engage in Bertrand competition. This means that the lowest cost provider of a good to a country will either use the optimal markup or, if necessary, set a limit price to just undercut the next-lowest-cost provider of the good.[15] Given the vector of scale parameters $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_n)$, a static equilibrium is given by a profile of wages $(w_1, \ldots, w_n)$ such that labor markets clear in all countries.

We now briefly present the basic equations that summarize the static trade equilibrium. Because the expressions for price indices, trade shares, and profit are identical to

[15]Note that we have assumed for simplicity that neither consumers nor workers internalize that their consumption or production decisions may affect the insights they may draw, and thus prices do not reflect the possibility that idea flows may result from the production or consumption of the good. This assumption is not innocuous; in general, prices and trading patterns depend on how much each agent internalizes.

Bernard et al. (2003), we relegate their derivation to Appendix A (Buera and Oberfield (2020)). In equilibrium, i's price index is

$$P_i \propto \left\{ \sum_j \lambda_j (w_j \kappa_{ij})^{-\theta} \right\}^{-1/\theta}.$$

The share of country i's expenditure that is spent on goods from country j, denoted by π_{ij}, can be expressed as

$$\pi_{ij} = \frac{\lambda_j (w_j \kappa_{ij})^{-\theta}}{\sum_{k=1}^{n} \lambda_k (w_k \kappa_{ik})^{-\theta}}.$$

Equilibrium wages depend on whether trade is balanced and where profit from producers is spent. Under the natural assumption that trade is balanced and that all profit from domestic producers is spent domestically, the labor market clearing conditions can be expressed as

$$w_j L_j = \sum_i \pi_{ij} w_i L_i.$$

Given the static equilibria, we next solve for the evolution of the profile of scale parameters $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_n)$ by specializing (4) for alternative assumptions about source distributions. We consider source distributions that encompass two cases: (i) new producers in a country learn from those that sell goods to the country, (ii) new producers learn from those that actively produce in the country.

2.1. *Learning From Sellers*

Following the framework introduced in Section 1, we model the evolution of technologies as the outcome of a process where producers combine "own ideas" with random insights from technologies in other sectors or countries. We first consider the case in which insights are drawn from sellers to the country. In particular, we assume that insights are randomly and uniformly drawn from the distribution of productivity among all producers that sell goods to a country.[16] In this case, the source distribution is given by

$$G_i(q) = G_i^S(q) \equiv \sum_j H_{ij}(q),$$

where $H_{ij}(q)$ is the fraction of goods for which the lowest cost provider of the good to i is a producer in j with productivity weakly less than q. As we show in Appendix A.3, after specializing equation (4) to this source distribution, the evolution of the scale of the

[16]For the case of learning from sellers, the assumption that insights are drawn uniformly from all sellers to the country is not central. Alternative assumptions, for example, insights are randomly drawn from the distribution of sellers' productivity in proportion either to consumption of each good or to expenditure on each good, give the same law of motion for the each country's stock of knowledge up to a constant. See Online Appendix E.4.

Fréchet distribution, that is, the stock of knowledge, is described by

$$\begin{aligned}\dot{\lambda}_{it} &= \alpha_{it}\int_0^\infty x^{\beta\theta}\, dG_i^S(q) \\ &= \Gamma(1-\beta)\alpha_{it}\sum_j \pi_{ij}\left(\frac{\lambda_j}{\pi_{ij}}\right)^\beta,\end{aligned} \tag{5}$$

where $\Gamma(\cdot)$ is the Gamma function.

Equation (5) connects the evolution of a country's stock of knowledge to the knowledge of its trading partners. It shows that trade shapes how a country learns in two ways. First, trade gives a country access to the ideas of sellers from other countries. Second, trade barriers affect which producers are able to sell goods to a country. On one hand, trade leads to tougher competition, so that there is more selection among the producers from which insights are drawn. Starting from autarky, lower trade barriers make it less likely that low-productivity domestic producers can compete with high-productivity foreign producers. The subsequent insights drawn from these high-productivity foreign producers will be better quality than those drawn from the low-productivity domestic producers.[17] Higher trade barriers, on the other hand, lead to more selection among foreign producers into selling goods to country i. In fact, the less a foreign country sells to country i, the stronger selection is among its producers. The average quality of insights drawn from j is given by $(\lambda_j/\pi_{ij})^\beta$, where λ_j/π_{ij} is an average of productivity among those in j that sell to i. Holding fixed j's stock of knowledge, a smaller π_{ij} reflects more selection into selling goods to i, which means that the insights drawn from sellers from j are likely to be higher quality insights.

The overall quality of insights is not necessarily maximized in the case of free trade. To optimize the quality of insights a country must bias its trade toward those countries with more knowledge. In particular, in the short run the growth of country i's stock of knowledge is maximized when its expenditure shares are proportional to its trading partners' stocks of knowledge:[18]

$$\frac{\pi_{ij}}{\pi_{ij'}} = \frac{\lambda_j}{\lambda_{j'}}. \tag{6}$$

In equilibrium, on the other hand, country i's expenditure shares will satisfy

$$\frac{\pi_{ij}}{\pi_{ij'}} = \frac{\lambda_j(w_j\kappa_{ij})^{-\theta}}{\lambda_{j'}(w_{j'}\kappa_{ij'})^{-\theta}}. \tag{7}$$

Notice that (6) and (7) coincide only if differences in trade costs perfectly offset differences in trading partners' wages. Suppose, for example, that trade costs are symmetric. If a country spends equally on imports from two trading partners, one with a high wage and one with a low wage, the country would improve the quality of its insights by tilting trade toward the trading partner with the higher wage. Intuitively, the marginal seller in the high wage country is more productive—and would generate higher quality insights—than the marginal seller in the low wage country, as the former must overcome the high

[17]This mechanism is emphasized by Alvarez, Buera, and Lucas (2013).

[18]This is the solution to $\max_{\{\pi_{ij}\}}\sum_j \pi_{ij}^{1-\beta}\lambda_j^\beta$ subject to $\sum_j \pi_{ij} = 1$.

wage. Similarly, a decline in the cost of trading with a low-wage country may be harmful as it might divert trade from a high wage country and lower the quality of insights.[19] Of course, whether free trade is optimal depends on what individuals are able to internalize; we have assumed that consumers do not internalize that their consumption decisions affect the quality of insights drawn by producers.[20]

2.2. *Learning From Producers*

Another natural source of ideas is the interaction of producers with other domestic producers, or workers employed by these producers. In this section, we consider the case in which the insights are drawn uniformly from the distribution of productivity among domestic producers that are actively producing.[21] We consider only the case in which trade costs satisfy the triangle inequality $\kappa_{jk} < \kappa_{ji}\kappa_{ik}$, $\forall i, j, k$ such that $i \neq j \neq k \neq i$. In this case, any producer that exports also sells domestically.[22] The source distribution is

$$G_i(q) = G_i^P(q) \equiv \frac{H_{ii}(q)}{H_{ii}(\infty)},$$

where, as before, $H_{ii}(q)$ is the fraction of goods for which the lowest cost provider to i is a domestic producer with productivity weakly less than q. As we show in Appendix A.3, specializing equation (4) to this source distribution, the evolution of a country's stock of

[19]To be clear, iceberg trade costs are not tariffs (which both distort trade costs and provide revenue), so the preceding argument does not show that the distorting trade represents optimal policy. However, if the shadow value of a higher stock of knowledge is positive, a planner that maximizes the present value of a small open economy's real income and can set country-specific tariffs would generically set tariffs that are nonzero and not uniform across trading partners.

[20]An interesting question for future research is how the predictions of the model would change if households internalized the insights they might gain when making consumption decisions. Similarly—and possibly more realistically—firms that draw insights from the intermediate inputs they use may internalize the value of these insights when making sourcing decisions. In the context of this model, however, this raises several technical obstacles. The main difficulty is that a consumer's willingness to pay for a good would depend on both the good's consumption value and the value of the insight the consumer might attain. The consumer's ranking of producers is likely to differ from the ranking of producers' costs of supplying her with the good. This means that she may not always buy from the seller that can provide the good at the lowest cost. It also means that the price charged by the preferred seller will incorporate both the difference in cost between the most- and second-most-preferred sellers and the difference in the value of insights. Aggregating and solving for the equilibrium decisions would become substantially more difficult. Further, solving for the trade equilibrium would no longer be a static problem. Because the value of the insight is a forward looking object, the relative weights on the consumption value and on the value of the insight would depend on the trajectory of the country's stock of knowledge.

How might trade patterns differ if households did, in fact, internalize the value of insights from consuming? While we have not solved such a model, we can speculate. Relative to our baseline, households would likely tilt their consumption bundles toward producers from countries with high wages and high trade costs, as those are the countries for whom the gap between the consumption value and the gains from the insight is largest.

[21]When insights are drawn from domestic producers, the assumption that insights are drawn uniformly, instead of in proportion to the labor used in the production of each good, is more important. See Online Appendix E.4 for a characterization of the dynamics of the stock of knowledge under alternative assumptions.

[22]To see this, suppose that there were a variety s such that i exports to j and k exports to i. This means that $\frac{w_i\kappa_{ji}}{q_i(s)} \leq \frac{w_k\kappa_{jk}}{q_k(s)}$ and $\frac{w_k\kappa_{ik}}{q_k(s)} \leq \frac{w_i\kappa_{ii}}{q_i(s)}$. Since $\kappa_{ii} = 1$, these imply that $\kappa_{ji}\kappa_{ik} \leq \kappa_{jk}$, a violation of the triangle inequality and thus a contradiction.

knowledge is described by

$$\dot{\lambda}_{it} = \alpha_{it} \int_0^{\infty} x^{\beta\theta} \, dG_i^P(q)$$
$$= \Gamma(1-\beta)\alpha_{it}\left(\frac{\lambda_i}{\pi_{ii}}\right)^{\beta}. \tag{8}$$

Thus, the source distribution of country i is a function of the share of its expenditure on domestic goods and the domestic stock of knowledge, λ_i.

How does trade alter a country's stock of knowledge? In autarky, insights are drawn from all domestic producers, including very unproductive ones. As a country opens up to trade the set of domestic producers improves as the unproductive technologies are selected out. This raises the quality of insights drawn and increases the growth rate of the stock of knowledge.[23]

2.3. *Other Specifications of Learning*

To this point, we have focused on two channels of idea flows. In the Online Appendix E, we explore several alternative ways of modeling the learning process. Online Appendix E.6 assumes that a producer would gain more and better insights if she were exposed to wider variety of production techniques. There we show that this extension has no impact for the case of learning from sellers but it can reverse the results when learning is from domestic producers. Online Appendix E.5 allows producers to focus their attention so that insights are drawn disproportionately from those that are more productive. For the case of learning from sellers, we derive the law of motion for stocks of knowledge with targeted learning and show that an environment with better targeting is quantitatively similar to an environment without targeting with higher β.

3. GAINS FROM TRADE

As in other gravity models, a country's real income and welfare can be summarized by its stock of knowledge (or some other measure of aggregate productivity), its expenditure share on domestic goods, and the trade elasticity, as described by Arkolakis, Costinot, and Rodríguez-Clare (2012):

$$y_i \equiv \frac{w_i}{P_i} \propto \left(\frac{\lambda_i}{\pi_{ii}}\right)^{1/\theta}. \tag{9}$$

In our model, gains from trade have a static and dynamic component. The static component, holding each country's stock of knowledge fixed, is the familiar gains from trade in standard Ricardian models, for example, Eaton and Kortum (2002). The dynamic gains from trade are the ones that operate through the effect of trade on the flow of ideas.

In this section, we focus on the determinants of the static and dynamic gains from trade. We first illustrate these using a simple example of a simultaneous change in trade barriers in a world with symmetric countries. We then return to the more general environment

[23]A version of this mechanism is emphasized by Sampson (2016) and Perla, Tonetti, and Waugh (2015). Perla, Tonetti, and Waugh (2015) had the additional feature that producers that drop out may upgrade their technology by imitating others.

with asymmetric countries and derive several analytical results that will be useful for understanding our quantitative results.[24]

As discussed before, to obtain growth in the long-run, we assume that the arrival rates of insights grow over time at rate γ, in which case it is convenient to analyze the evolution of detrended stocks of knowledge $\hat{\lambda}_{it} = \lambda_{it} e^{-\frac{\gamma}{1-\beta}t}$. On a balanced growth path, these solve the system of nonlinear equations

$$\text{Sellers}: \quad \hat{\lambda}_i = \frac{(1-\beta)\Gamma(1-\beta)}{\gamma} \hat{\alpha}_i \sum_{j=1}^{n} \pi_{ij}^{1-\beta} \hat{\lambda}_j^{\beta}, \tag{10}$$

$$\text{Producers}: \quad \hat{\lambda}_i = \frac{(1-\beta)\Gamma(1-\beta)}{\gamma} \hat{\alpha}_i \left(\frac{\hat{\lambda}_i}{\pi_{ii}}\right)^{\beta} \propto \hat{\alpha}_i^{\frac{1}{1-\beta}} \pi_{ii}^{-\frac{\beta}{1-\beta}}. \tag{11}$$

Equations (10) and (11) are sufficient to characterize the gains from trade relative to autarky in terms of "observables" of the current equilibrium, that is, stocks of knowledge and trade shares. Let y_i^{aut} and λ_i^{aut} be what i's real income and stock of knowledge would be in autarky. These are related by $\frac{y_i}{y_i^{\text{aut}}} = (\frac{\lambda_i/\pi_{ii}}{\lambda_i^{\text{aut}}})^{1/\theta}$. For either specification of learning, the stock of knowledge in autarky would solve $\hat{\lambda}_i^{\text{aut}} = \frac{(1-\beta)\Gamma(1-\beta)}{\gamma} \hat{\alpha}_i (\hat{\lambda}_i^{\text{aut}})^{\beta}$. This along with (10) and (11) implies that along a balanced growth path:

$$\text{Sellers}: \quad \frac{\lambda_i}{\lambda_i^{\text{aut}}} = \left(\sum_{j=1}^{n} \pi_{ij}^{1-\beta} \left(\frac{\lambda_j}{\lambda_i}\right)^{\beta}\right)^{\frac{1}{(1-\beta)}}, \tag{12}$$

$$\text{Producers}: \quad \frac{\lambda_i}{\lambda_i^{\text{aut}}} = \pi_{ii}^{-\frac{\beta}{1-\beta}}. \tag{13}$$

For either specification of learning, the gains relative to autarky can be quite large, especially when β is close to one. It is also possible, in the learning from sellers specification, that a country's stock of knowledge is lower than it would have been in autarky. Nevertheless, the total gains relative to autarky are always positive.[25]

3.1. *Gains From Trade in a Symmetric Economy*

Consider a world with n symmetric countries in which there is a common iceberg cost κ of shipping a good across any border. In such a world, the share of a country's expenditure on domestic goods is $\pi_{ii} = \frac{1}{1+(n-1)\kappa^{-\theta}}$, while the share of its expenditure on imports from each trading partner is $\frac{1-\pi_{ii}}{n-1}$. Each country's detrended real per-capita income is obtained

[24] In the Online Appendix H.3, we explore an example with a richer geography in which trade barriers generate a core-periphery structure. We examine how the dynamic gains from trade determines the gap in income between core and periphery countries.

[25] To see that the total gains from trade are always positive, note that (12) can be rearranged as $\frac{\lambda_i}{\lambda_i^{\text{aut}}} = (\pi_{ii}^{1-\beta} + \sum_{j \neq i} \pi_{ij}^{1-\beta} (\frac{\lambda_j}{\lambda_i})^{\beta})^{\frac{1}{(1-\beta)}} \geq (\pi_{ii}^{1-\beta})^{\frac{1}{(1-\beta)}} = \pi_{ii}$. This implies that $\frac{y_i}{y_i^{\text{aut}}} = (\frac{\lambda_i/\pi_{ii}}{\lambda_i^{\text{aut}}})^{1/\theta} \geq 1$. For country i, dynamic losses relative to autarky can occur if there is a j with π_{ij} close to 1 and $\lambda_j \ll \lambda_i$, which can be consistent with equilibrium if $L_j \gg L_i$ and $\alpha_j \ll \alpha_i$. However, π_{ij} close to 1 is exactly the case where the static gains are large, so even in this case the total gains from trade are positive. In our quantitative exercise, we only find dynamic losses for the case of Switzerland.

by specializing either (10) or (11) to the symmetric economy and substituting the resulting stock of knowledge into (9):

$$\text{Sellers}: \quad \hat{y}_i \propto \pi_{ii}^{-\frac{1}{\theta}} \hat{\lambda}^{\frac{1}{\theta}} \propto \pi_{ii}^{-\frac{1}{\theta}} \left[\pi_{ii}^{1-\beta} + (n-1) \left(\frac{1-\pi_{ii}}{n-1} \right)^{1-\beta} \right]^{\frac{1}{1-\beta}\frac{1}{\theta}}, \tag{14}$$

$$\text{Producers}: \quad \hat{y}_i \propto \pi_{ii}^{-\frac{1}{\theta}} \hat{\lambda}^{\frac{1}{\theta}} \propto \pi_{ii}^{-\frac{1}{\theta}} \pi_{ii}^{-\frac{\beta}{1-\beta}\frac{1}{\theta}} = \pi_{ii}^{-\frac{1}{1-\beta}\frac{1}{\theta}}. \tag{15}$$

The terms $\pi_{ii}^{-\frac{1}{\theta}}$ and $\hat{\lambda}^{\frac{1}{\theta}}$, respectively, correspond to the static and dynamic gains from trade. Both depend on the curvature of the productivity distribution, θ; a higher θ corresponds to thinner right tails. With higher θ, there are fewer highly productive producers abroad whose goods can be imported, and there are fewer highly productive producers from whom insights may be drawn. The novel parameter determining the gains from trade is β. The parameter β controls the importance of insights from others in the quality of new ideas, and hence the extent of technological spillovers associated with trade. With higher β, insights from others are more important and, therefore, more is gained by being exposed to more productive producers. This can be seen most clearly by comparing autarky to costless trade. Equations (14) and (15) reveal that for either specification of learning, the ratio of real income under costless trade (with $\pi_{ii} = 1/n$) to real income under autarky (with $\pi_{ii} = 1$) is $n^{\frac{1}{1-\beta}\frac{1}{\theta}}$. This is increasing in β and grows arbitrarily large as β approaches 1.[26] As in many trade models, the gains from specialization increase as the number of countries grows large. Here, these static gains bring with them better quality insights.

Using these equations, we can ask how a change in trade costs would impact countries' real incomes. For several values of β, the top panels of Figure 1 illustrate the common value of each country's stock of knowledge relative to its level under free trade.[27] The bottom panels show the corresponding real income per capita. The left (right) panels focus on the specification of learning in which insights are drawn from sellers (domestic producers). As a benchmark, the dotted line represents $\beta = 0$, which corresponds to the static trade model of Eaton and Kortum (2002). As trade costs rise, countries become more closed and their stocks of knowledge decline. When β is larger, the dynamic gains from trade are larger.

To interpret these figures, it is instructive to contrast the total gains from trade under each specification of learning with the static gains. For each, we can summarize the change

[26]These limiting cases are close to the models analyzed by Alvarez, Buera, and Lucas (2013), Sampson (2016), and Perla, Tonetti, and Waugh (2015). When $\beta = 1$, the steady state gains from moving from autarky to free trade are infinite because integration raises the growth rate of the economy. In contrast, for any $\beta < 1$, integration raises the level of incomes but leaves the growth rate unchanged.

[27]In this numerical example, we consider a world with $n = 50$ economies with symmetric populations, so that each country is of the size of Canada or South Korea. We set the shape parameter of the Fréchet distribution to $\theta = 4$. This value is in the range consistent with estimates of trade elasticities. See Simonovska and Waugh (2014) and the references therein. Given a value of β, the growth rate of the arrival rate of ideas is calibrated so that on the balanced growth path each country's TFP grows at 1%, $\frac{\gamma}{(1-\beta)\theta} = 0.01$. The parameter $\hat{\alpha}$ is normalized so that in the case of costless trade, $\kappa_n = 1$, the detrended stock of knowledge equals 1.

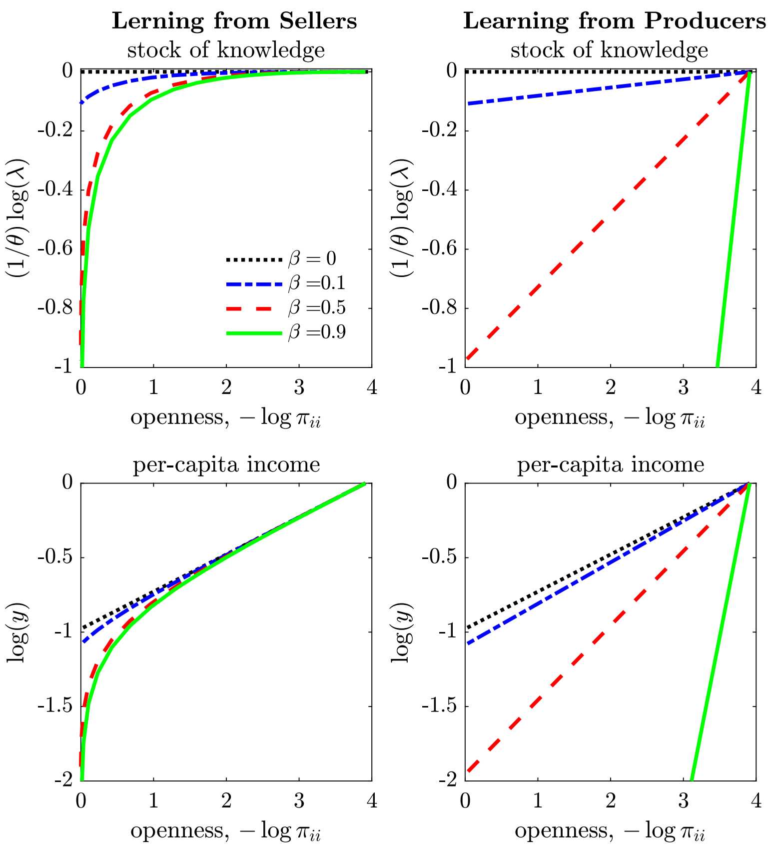

FIGURE 1.—Gain From Reducing Common Trade Barriers. **Note**: This figure shows each country's stock of knowledge and per-capita income relative to their values under costless trade. See footnote 27 for additional details of the calibration used in this figure.

in real income in terms of a multiplier of the change in the domestic expenditure share.[28]

$$\text{Sellers}: \quad d\ln \hat{y}_i = -\frac{1}{\pi_{ii}^{1-\beta}\left(\dfrac{1-\pi_{ii}}{n-1}\right)^{\beta} + (1-\pi_{ii})}\frac{1}{\theta}\,d\ln \pi_{ii}, \tag{16}$$

$$\text{Producers}: \quad d\ln \hat{y}_i = -\frac{1}{1-\beta}\frac{1}{\theta}d\ln \pi_{ii}. \tag{17}$$

[28]A convenient feature of the symmetric example is that, since every country has the same stock of knowledge and the same wage, the share of a country's expenditure on domestic goods is $\frac{1}{1+(n-1)\kappa^{-\theta}}$. Thus the change in κ causes the same change in trade shares whether stocks of knowledge are held fixed, insights are drawn from sellers, or insights are drawn from producers. In a world with asymmetric countries, a change in trade barriers (e.g., κ_{ij}) would cause different changes in trade shares in each version of the model. However, as we will show below, the overarching message—that when insights are drawn from producers the dynamic gains amplify the static gains whereas when insights are drawn from sellers the dynamics gains are largest when countries are close to autarky—will remain.

The usual expression for the static gains from trade, $d\ln y_i = -\frac{1}{\theta}d\ln\pi_{ii}$ can be found by setting $\beta = 0$.

When insights are drawn from sellers, the dynamic gains from reducing trade barriers are qualitatively different from the static gains. The dynamic gains are largest when the world is relatively closed, whereas the static gains are largest when the world is relatively open. This can be seen from the left panels of Figure 1 but also by inspecting the limiting values of (16). As the world becomes more open, the total gains from reducing trade barriers corresponds to the static gains, $\lim_{\pi_{ii}\to 1/n}\frac{d\ln\hat{y}_i}{d\ln\pi_{ii}} = -\frac{1}{\theta}$. In contrast, as the world becomes more closed, the marginal dynamic gains grow arbitrarily large, $\lim_{\pi_{ii}\to 1}\frac{d\ln\hat{y}_i}{d\ln\pi_{ii}} = -\infty$.[29] Put differently, when the economy is relatively open, the total gains from reducing trade barriers are composed mostly of the static gains, whereas when the world is relatively closed, the total gains are composed mostly of the dynamic gains.

To understand this, consider a country close to autarky. If trade costs decline, the marginal import tends to be made by a foreign producer with high productivity. While the high trade costs imply that the static gains from trade remain relatively small, the insights drawn from this marginal producer tends to be of high quality. In contrast, for a country close to free trade, the reduction in trade costs leads to large infra-marginal static gains from trade, but the insights drawn from the marginal producers are likely to be of lower quality.

In contrast, when insights are drawn from domestic producers, the dynamic gains from reducing trade barriers simply amplify the static gains. When trade barriers decline, fewer low-productivity domestic producers find it profitable to operate. The insights from those producers are replaced by a proportionate increase in insights from all producers that continue to operate. This argument is the same regardless of the current level of openness. Consequently, as with static gains, the dynamic gains are log-linear in a country's own-trade share, $\hat{\lambda}_i \propto \pi_{ii}^{-\frac{\beta}{1-\beta}}$, as shown clearly in the bottom right panel.

The diffusion parameter β determines the extent to which learning amplifies the static gains from trade. One way of interpreting equation (17) is that the diffusion of ideas causes the static gains from trade to compound itself. The expression for the static and dynamic gains from trade shares features with an analogous expression in a static world in which production uses intermediate inputs.[30]

3.2. *The Pure Diffusion Limit:* $\beta \nearrow 1$

In this section, we return to an environment with asymmetric countries and trade costs and study the limiting economy as $\beta \nearrow 1$, a limit that is particularly revealing about the gains from trade in the learning-from-sellers specification. Our main result for the learning from sellers specification is that when a country's trade shares are interior, catch-up growth resulting from a change in trade barriers is small when β is either close to zero or close to one.

[29]The argument that $\lim_{\pi_{ii}\to 1}\frac{d\ln\hat{y}_i}{d\ln\pi_{ii}} = -\infty$ does not rely on symmetry. In fact, in an asymmetric world, the marginal gains from opening to any new trade partner are infinite: $\lim_{\pi_{ij}\to 0}\frac{d\ln\hat{y}_i}{d\ln\pi_{ij}} = -\infty$ for $j \neq i$.

[30]In a world with roundabout production, a decline in trade costs reduces the costs of production, lowering the cost of intermediate inputs, which lowers the cost of production further, etc. Here, when trade costs decline, producers draw better insights from others, raising stocks of knowledge, and this improves the quality of insights others draw, etc. The parameter β gives the contribution of an insight to a new idea, just as the share of intermediate goods measures the contribution of the cost of intermediate inputs to marginal cost.

Suppose that some iceberg trade cost κ_{lm} changes. If β equals zero, then by construction there are no dynamic gains from trade. For the limiting economy as $\beta \nearrow 1$, the marginal dynamic gains resulting from lowering any trade barrier are equal for all countries, as long as trade shares are interior[31]

$$\lim_{\beta \nearrow 1} \frac{d \ln \hat{\lambda}_i}{d\kappa_{lm}} = \frac{\displaystyle\sum_j \sum_k \hat{\lambda}_j \hat{\lambda}_k \frac{d \ln \pi_{jk}}{d\kappa_{lm}}}{\displaystyle\sum_j \sum_k \hat{\lambda}_j \hat{\lambda}_k}. \tag{18}$$

Thus even if i's stock of knowledge is much lower than that of its trading partners, lowering its trade barriers with other more productive countries will not reduce the gap in knowledge, that is, the dynamic gains from catching up to other countries are zero. This may seem puzzling; as the contribution of insights from others in the productivity of new ideas becomes larger and the model approaches one of pure diffusion, marginal catch-up growth becomes relatively unimportant.

It is useful to contrast this with the gains relative to autarky. Equation (12) relates a country's stock of knowledge to what it would have been in autarky. Given current stocks of knowledge and current trade shares, the implied gains relative to autarky are much larger when β is close to 1:

$$\lim_{\beta \nearrow 1} \lambda_i / \lambda_i^{\text{aut}} = \infty.$$

To summarize, if an economy is moderately open, the catchup growth from lowering trade costs are small if the model is close to one of pure innovation ($\beta = 0$) or close to one of pure diffusion ($\beta \nearrow 1$). Those two models differ, however, in the gains relative to autarky. In an environment close to pure diffusion, when a country moves from autarky to only slightly open, the dynamic gains from trade are quite large. But any subsequent lowering of trade costs has a relatively small impact on the country's stock of knowledge. It is only for intermediate values of β that lowering trade barriers would have a larger dynamic impact on a country's stock of knowledge for a wide range of trade shares.

Why is catch-up growth concentrated near autarky when β is close to one? The strength of diffusion indexes the diminishing returns in the contribution of the quality of an insight to the productivity of a new idea. When β is small, there are strong diminishing returns, and the quality of insights drawn from others has little impact on the productivity of any new idea. When β is larger, higher quality insights make larger contributions to new ideas. When β is close to one, the difference in ideas coming from high- and low-quality insights becomes so large that the economy becomes dominated by ideas generated from the highest quality insights.[32] When a country is only slightly open, it is already importing goods

[31]We take this limit holding fixed the stocks of knowledge $\{\hat{\lambda}\}$. To see that, when trade shares are interior, that dynamic gains from trade are the same for each country, note that differentiating (10) implies that the change in i's stock of knowledge satisfies $\frac{d \ln \hat{\lambda}_i}{d\kappa_{lm}} = \sum_j \Omega_{ij}[(1-\beta)\frac{d \ln \pi_{ij}}{d\kappa_{lm}} + \beta \frac{d \ln \hat{\lambda}_j}{d\kappa_{lm}}]$, where $\Omega_{ij} \equiv \frac{\pi_{ij}^{1-\beta} \hat{\lambda}_j^{\beta}}{\sum_k \pi_{ik}^{1-\beta} \hat{\lambda}_k^{\beta}}$. Taking the limit of both sides and noting that $\lim_{\beta \nearrow 1} \Omega_{ij} = \frac{\hat{\lambda}_j}{\sum_k \hat{\lambda}_k}$ gives $\lim_{\beta \nearrow 1} \frac{d \ln \hat{\lambda}_i}{d\kappa_{lm}} = \frac{\sum_j \hat{\lambda}_j (\lim_{\beta \nearrow 1} \frac{d \ln \hat{\lambda}_j}{d\kappa_{lm}})}{\sum_j \hat{\lambda}_j}$, which is independent of i. The remainder of the derivation of (18) can be found in the Online Appendix C.2.

[32]The role of β in modulating the contribution of the highest quality insights is closely related to the role of returns to scale in an economy with heterogeneous firms, such as Lucas, Jr. (1978). An economy in which the returns to scale of production functions is close to constant becomes dominated by those firms with the highest productivity.

from most of the highest productivity foreign producers. Indeed, as $\beta \nearrow 1$, if a country is even slightly open, its stock of knowledge relative to its trade partners is the same as it would be under costless trade. Further opening brings new insights that tend to be just a bit worse and are, in this limit, irrelevant for further catch-up growth.[33,34] This feature of the model will be especially important for understanding our quantitative results when we study the implications of actual changes in trade volumes.

4. QUANTITATIVE EXPLORATION

We now explore the ability of the theory to account for the evolution of the distribution of productivity across countries in the post-war period. In particular, after calibrating preferences and technologies, we choose the evolution of bilateral trade costs and country-specific arrival rate of ideas so that the model exactly matches the observed evolution of bilateral trade and measured TFP.[35] We then quantify how much of world TFP growth and the variation of TFP growth in the 1962–2000 period can be accounted for by the measured changes in trade costs.

With this in mind, we extend the simple trade model introduced in Section 2 to incorporate intermediate inputs, nontraded goods, and a broader notion of labor which we refer to as equipped labor. In addition, we focus on the case in which insights are drawn from sellers to a market. This version of the model has particularly rich testable implications and, as we show in this section, it provides a promising quantitative theory of dynamic gains from trade.[36] At the end of this section, we discuss results for three alternative specifications of learning.

4.1. *Extended Trade Model*

Suppose now that a producer of good s in country i with productivity q has access to a constant returns to scale technology combining an intermediate input aggregate (x) and equipped labor (l)

$$y_i(s) = \frac{1}{\eta^\eta (1-\eta)^{1-\eta}} q x_i(s)^\eta l_i(s)^{1-\eta}.$$

All goods use the intermediate input aggregate, or equivalently, the same bundle of intermediate inputs. The intermediate input aggregate is produced using the same technology

[33] Alvarez, Buera, and Lucas (2013) analyzed an economy similar to the limit point $\beta = 1$. In particular, their Propositions 7 and 8 show that the behavior of the tail of the distribution of productivity is independent of trade costs, as long as trade costs are finite.

[34] Inspecting (5) reveals that trade shares affect the law of motion for a country's stock of knowledge in two ways: as weights that add up the contributions of insights from trading partners and as measuring the selection effect that when trade barriers are larger, the most productive firms are much more likely to export. When β is larger, the impact of selection on the quality of new ideas is larger because the quality of insights matter more. Indeed as $\beta \nearrow 1$, the two effects actually cancel, so that the contribution of insights from each trading partner becomes independent of trade shares. Note that this argument goes through only if the trade shares are interior. In the limit, the contribution of insights from trading partner is invariant to the trade share as long as the trade share is strictly positive. In other words, the limit as $\beta \nearrow 1$ is discontinuous in π_{ij} at $\pi_{ij} = 0$.

[35] Measured TFP refers to the aggregate Solow residual, that is, real aggregate output net of the contribution of aggregate physical capital and quality adjusted aggregate labor inputs. For brevity, we will use the term TFP.

[36] In the Online Appendix G, we present reduced-form evidence consistent with the mechanisms emphasized by this version of the model: both openness and trade with more productive countries are associated with higher productivity growth.

as the consumption aggregate, so that the market clearing condition for intermediate inputs for i is

$$\int x_i(s)\,ds = \left[\int \chi_i(s)^{\frac{\varepsilon-1}{\varepsilon}}\,ds\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\chi_i(s)$ denotes the amount of good s used in the production of the intermediate input aggregate. Equipped labor L is produced with an aggregate Cobb–Douglas technology requiring capital and efficiency units of labor[37]

$$L_i = \int l_i(s)\,ds = \frac{1}{\zeta^{\zeta}(1-\zeta)^{1-\zeta}} K_i^{\zeta}(h_i\tilde{L}_i)^{1-\zeta}.$$

In our quantitative exercises, we take an exogenous path of aggregate physical and human capital, K_i and h_i, from the data. We thus abstract from modeling the accumulation of these factors, and hold them constant in counterfactuals.

In addition to the iceberg transportation costs κ_{ij}, we assume that a fraction $1-\mu$ of the goods are nontradable, that is, this subset of the goods face infinite transportation costs.[38] One effect of introducing nontraded goods is that in the extended model the value of the elasticity of substitution ε affects equilibrium allocations. In Appendix B, we present the expressions for price indices, trade shares, and the evolution of stocks of knowledge for this version of the model.

4.2. *Calibration*

We need to calibrate seven common parameters, $(\theta, \eta, \zeta, \mu, \gamma, \varepsilon, \beta)$, and two sets of parameters that are country and time specific, the matrix of transportation costs $\mathbf{K}_t = [\kappa_{\text{int}}]$ and the vector of arrival rates $\boldsymbol{\alpha}_t = (\alpha_{1t}, \dots, \alpha_{nt})$.

We set $\theta = 4$. This value is in the range consistent with estimates of trade elasticities.[39] We choose $\zeta = 0.36$ to match the corporate labor share in the US in 2000 calculated by Karabarbounis and Neiman (2014). We set $\eta = 0.48$ to match the share of intermediate inputs in gross output for the world economy in 2000, according to the World Input Output Database (WIOD). We set the share of tradable goods $\mu = 0.34$ to match the fraction of agriculture, mining, and manufacturing in gross output in 2000 in the WIOD. We set $\varepsilon = 1$, but note that alternative values do not affect the results significantly.

Following the strategy in Waugh (2010), we show in Appendix B that given values for θ, μ, and ε as well as data on bilateral trade shares and relative prices over time, the iceberg

[37]Implicitly, we are assuming that individual technologies are

$$y(s) = \frac{1}{\eta^{\eta}(1-\eta)^{1-\eta}\zeta^{(1-\eta)\zeta}(1-\zeta)^{(1-\eta)(1-\zeta)}} q x(s)^{\eta}\big[k(s)^{\zeta}\big(h_i l(s)\big)^{1-\zeta}\big]^{1-\eta}$$

and that investment can be produced with the same technology as the consumption and intermediate input aggregates.

[38]While these goods are not traded, we assume that producers of these goods may gain insights from other producers in the economy. In the Online Appendix E.2, we explore an alternative model in which idea flows do not affect the productivity of the nontradable sector.

[39]See Simonovska and Waugh (2014) and the references therein.

cost of shipping a tradable good to country i from country j at time t is

$$\kappa_{ijt} = \frac{p_{it}}{p_{jt}}\left(\frac{1-\pi_{iit}}{\pi_{ijt}}\frac{Z_{it}}{1-Z_{it}}\right)^{\frac{1}{\theta}}\left[\frac{(1-\mu)+\mu Z_{it}^{-\frac{\varepsilon-1}{\theta}}}{(1-\mu)+\mu Z_{jt}^{-\frac{\varepsilon-1}{\theta}}}\right]^{\frac{1}{\varepsilon-1}},$$

where $Z_{it} = (p_i^\eta w_i^{1-\eta})^{-\theta}\lambda_i/[\sum_j (p_j^\eta w_j^{1-\eta}\kappa_{ij})^{-\theta}\lambda_j]$, the share of i's expenditure on tradable goods spent on domestic tradables, and solves[40]

$$\pi_{iit} = \frac{(1-\mu)+\mu Z_{it}^{1-\frac{\varepsilon-1}{\theta}}}{(1-\mu)+\mu Z_{it}^{-\frac{\varepsilon-1}{\theta}}}.$$

Notice that if $\mu = 1$ we regain the standard implications of Ricardian models with Fréchet distributions in which allocations are invariant to the elasticity of substitution across varieties.

To assign values to the vector of arrival rates $\hat{\boldsymbol{\alpha}}_t = (\hat{\alpha}_{1t}, \dots, \hat{\alpha}_{nt})$ we proceed in two steps. Given the evolution of trade flows summarized by Z_{it}, we compute, in each year, the stocks of knowledge needed to match each country's TFP using

$$\hat{\lambda}_{it} \propto \left[(1-\mu)+\mu Z_{it}^{-\frac{\varepsilon-1}{\theta}}\right]^{-\frac{\theta}{\varepsilon-1}}\left(\frac{w_{it}}{p_{it}}\right)^{(1-\eta)\theta}.$$

This is a generalization of equation (9) for the model with intermediate inputs and nontraded goods. We measure TFP in the data as a standard Solow residual using real GDP, physical capital (K), employment (emp), and average human capital (h) from the PWT 8.0, that is, $TFP = \text{real GDP}/[K^\zeta \cdot (emp \cdot h)^{(1-\zeta)}]$.[41] To operationalize these equations, we use bilateral trade data for 1962–2000 from Feenstra, Lipsey, Deng, Ma, and Mo (2005) and data on real GDP and the price index from PWT 8.0 (Feenstra, Inklaar, and Timmer (2015)).[42]

Given the evolution of trade flows and stocks of knowledge as well as values for β and γ, we back out sequences of arrival rates of ideas using the law of motion of stocks of

[40] The term $Z_i^{-1/\theta}$ is also the price index of nontradables relative to that of tradable goods in i.

[41] In any calibration of the model, we must take a stand on how to apportion a country's TFP into a stock of knowledge, which may generate idea flows, and other factors, such as allocational efficiency, that are unlikely to diffuse across borders. Our baseline calibration assumes that physical and human capital differences are unlikely to diffuse across borders, but that after controlling for those, all residual TFP differences are due to differences in the stocks of knowledge and trade barriers. In the Online Appendix E.1, we consider an alternative calibration strategy. We project $\log TFP$ onto R&D intensity, the log of the human capital stock and the log of an import-weighted average of trading partners' TFP. We assign the residual TFP from this regression to a neutral productivity terms affecting the units of equipped labor and not the stock of knowledge, and choose stocks of knowledge to match predicted TFP from the regression. There we show that the results in this section are robust to the alternative calibration strategy.

[42] In particular, we measure real GDP using real GDP at constant national prices (rgdpna). We scale the real GDP series for each country so that its value in 1962 coincides with the real GDP measure given by the expenditure-side real GDP at chained PPP (cgdpe). We measure the price index using the price level of cgdpe (pl_gdpe). We identify expenditure-side real GDP at chained PPP in the data with real income, which is proportional to wL/p in our model. Relatedly, we identify the price level of the expenditure-side real GDP at chained PPP in the data with the ideal price index in our model, p. Similar results are obtained if we use output side variables or if we choose parameters that make PPP in the model coincide with PPP in the data, as explained in the Online Appendix E.8.

knowledge

$$\hat{\lambda}_{it+1} \propto \hat{\alpha}_{it} \left\{ (1-\mu)\hat{\lambda}_{it}^{\beta} + \mu \left[Z_{it}^{1-\beta}\hat{\lambda}_{it}^{\beta} + \sum_{j\neq i} \left(\frac{\pi_{ijt}}{\frac{1-\pi_{iit}}{1-Z_{it}}} \right)^{1-\beta} \hat{\lambda}_{jt}^{\beta} \right] \right\}$$
$$+ \left(1 - \frac{\gamma}{1-\beta}\right)\hat{\lambda}_{it}. \tag{19}$$

This is a discrete time generalization of (5) for the extended model. The sequence of arrival rates of ideas are the residuals needed to explain the evolution of TFP between 1962 and 2000 given the dynamics of trade costs.[43]

We are left with two parameters to calibrate: the strength of the diffusion, β, and the growth rate of the arrival rate of ideas, γ. Identifying the growth rate of the arrival of ideas with the average growth rate of population in the US between 1962 and 2000 gives $\gamma = 0.01$. We then find the β so that the implied sequence of arrival rates for the US is consistent with γ. That is, given $\gamma = 0.01$ and any value of β, there is a sequence of arrival rates that satisfies (19). We choose the β so that the average growth rate of those implied arrival rates in the US between 1962–2000 is $\gamma = 0.01$. We take this value, $\beta = 0.6$, to be our benchmark.[44]

In addition, we explore how well the model can quantitatively account for cross-country income differences and the evolution of countries' productivity over time for alternative values of $\beta \in (0, 1)$. When we consider alternative values of the diffusion parameter β, we recalibrate γ in an analogous way as a fixed point: we find the value of γ such that the average growth rate of the implied arrival rates from (19) for the US between 1962–2000 is equal to γ.[45]

4.3. *Sample Selection*

The sample of countries in our quantitative analysis consists of a balanced panel of countries that is obtained by merging the PWT 8.0 with the NBER-UN dataset on bilateral trade flows from 1962 to 2000. We further restrict this sample to those countries with a population above 1 million in 1962 and oil rents that are smaller than 20% of GDP in 2000. We exclude Hong Kong, Malaysia, Panama, and Singapore, as these are countries where reexports play a very large role. The final sample consists of 64 countries.[46]

[43]When implementing this procedure we restrict the sequence of α_{it} to be nonnegative. In the cases where a negative α_{it} is required, we adjust the TFP and the equipped labor so that $\alpha_{it} = 0$ exactly matches the data. We obtain similar results if we let α_{it} take negative values.

[44]In the Online Appendix I.1, we report the evolution of the average calibrated trade costs and the evolution of calibrated arrival rates of ideas for the four miracle economies that we feature in Section 4.4.1.

[45]To rationalize the trade imbalances we observe in the data, we choose time-varying and pair-specific transfers, which are not changed in our counterfactuals exercises.

[46]Argentina, Australia, Austria, Belgium–Luxemburg (we consider the sum of the two countries, as the UN-NBER trade data is reported only for the sum), Bolivia, Brazil, Cameroon, Canada, Chile, China, Colombia, Costa Rica, Cote d'Ivoire, Denmark, Dominican Republic, Ecuador, Egypt, Finland, France, Germany, Ghana, Greece, Guatemala, Honduras, India, Indonesia, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kenya, South Korea, Mali, Mexico, Morocco, Mozambique, Netherlands, New Zealand, Niger, Norway, Pakistan, Paraguay, Peru, Philippines, Portugal, Senegal, South Africa, Spain, Sri Lanka, Sweden, Switzerland, Syria, Taiwan, Tanzania, Thailand, Tunisia, Turkey, Uganda, United Kingdom, United States, Uruguay, and Zambia.

4.4. *Explaining the Dynamics of TFP*

This section studies the ability of the model to account for the evolution of productivity over time. We ask: what is the growth of TFP that can be be attributed to measured changes in trade costs?

As discussed in Section 4.2, we use expenditure shares to back out the evolution of bilateral iceberg trade costs over time. We use the shorthand $\boldsymbol{\alpha}_t$ and $\boldsymbol{\kappa}_t$ to denote the sequence of vectors of arrival rates and matrices of trade costs that are required to match the data. To assess the contribution of trade, we compute several counterfactuals. We first find a baseline counterfactual in which each country's TFP is what it would have been had the world remained on a balanced growth path since 1962—the vector of TFP in each year would be a scalar multiple of the vector of 1962 TFPs. In this counterfactual, trade costs and (detrended) arrival rates $\{\hat{\alpha}_i\}$ remain constant at the level would that would be consistent with the cross-section in 1962. We use the shorthand $\boldsymbol{\alpha}_0$ and $\boldsymbol{\kappa}_0$ to denote the sequence of vectors of arrival rates and matrices of trade costs in this counterfactual.

We compute two counterfactuals to assess the gains from trade, that is, the contribution of trade to changes in TFP. Each provides a different way of dividing changes in each country's TFP into a contribution from changes in trade barriers and a contribution from changes in the arrival rates of ideas.[47] First, we compute how countries' TFP would have evolved if trade costs evolved as they do in data but each country's arrival rate of ideas remained fixed at its 1962 level. The second counterfactual computes the changes in TFP if the arrival rates of ideas evolved as they do in the data but the trade costs remained fixed at their 1962 levels.

These two counterfactuals provide two decompositions of observed changes in TFP that is summarized by the following two equations:[48]

$$\ln \frac{TFP_i(\alpha_t, \kappa_t)}{TFP_i(\alpha_0, \kappa_0)} = \underbrace{\ln \frac{TFP_i(\alpha_0, \kappa_t)}{TFP_i(\alpha_0, \kappa_0)}}_{\text{gains from trade 1}} + \underbrace{\ln \frac{TFP_i(\alpha_t, \kappa_t)}{TFP_i(\alpha_0, \kappa_t)}}_{\text{cont. from arrival rates}}, \tag{20}$$

$$\ln \frac{TFP_i(\alpha_t, \kappa_t)}{TFP_i(\alpha_0, \kappa_0)} = \underbrace{\ln \frac{TFP_i(\alpha_t, \kappa_0)}{TFP_i(\alpha_0, \kappa_0)}}_{\text{cont. from arrival rates}} + \underbrace{\ln \frac{TFP_i(\alpha_t, \kappa_t)}{TFP_i(\alpha_t, \kappa_0)}}_{\text{gains from trade 2}}. \tag{21}$$

Each decomposition involves two terms. One measures the gains from trade, while the remainder measures the contribution of changes in the arrival rate of ideas.[49]

4.4.1. *Growth Miracles*

To illustrate the dynamic gains from trade more concretely, we begin by studying the model's predicted changes in TFP during several growth miracles.

[47]Of course, changes in trade costs may themselves affect incentives to innovate, and hence the arrival of ideas. In the Online Appendix D, we extend the result of Eaton and Kortum (2001) that on any balanced growth path, each country's research effort is independent of trade barriers (although research effort may vary due to other things, such as taxes). Thus over long periods of time, treating the arrival rate of ideas as independent of trade costs may be a good approximation.

[48]We use here the shorthand $TFP_{it}(\alpha, \kappa)$ to indicate the TFP of country i in year t in a counterfactual with the sequence of vectors of arrival rates α and the sequence of matrices of trade costs κ and initial stocks of knowledge λ_{i1962} matching our calibration.

[49]This is in some ways analogous to dividing changes in nominal GDP into changes in a price index and changes in a quantity index. If the price index is a Lespeyres index, then the quantity index is a Paasche index and vice versa.

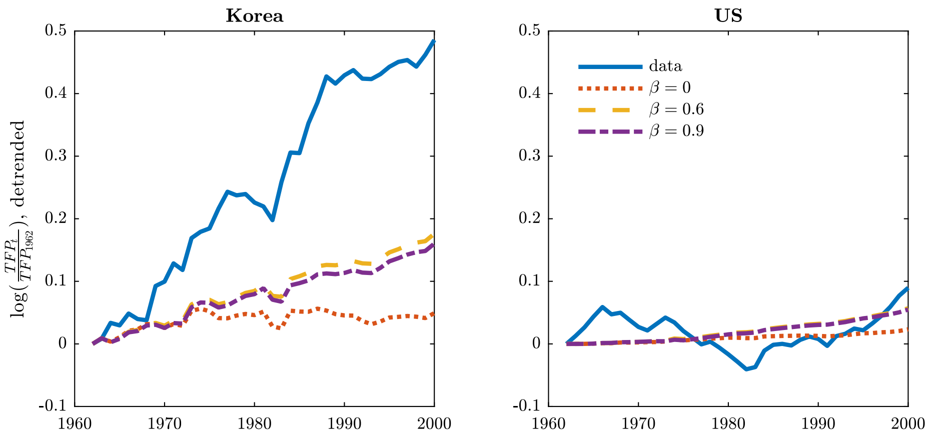

FIGURE 2.—Openness and the Evolution of TFP: South Korea and the US. **Note**: This figure plots the changes in detrended TFP for South Korea (left panel) and the US (right panel) under the specification of learning from sellers. In each panel, we plot the actual change in TFP and changes in TFP generated by the model when only trade costs change for three values of the diffusion parameter, $\beta = 0, 0.6, 0.9$. In all cases, TFP is detrended by the trend growth rate of TFP along a balanced growth path, $\gamma/(\theta(1-\beta))$. For each value of β, γ is recalibrated.

We first contrast the implied evolution of TFP in South Korea and the US. South Korea is a particularly interesting example as it is one of the most successful growth miracles in the post-war period, and a country that became much more integrated with the rest of the world, as inferred from the behavior of trade flows. The US economy provides a natural benchmark developed economy.[50]

Figure 2 explores the evolution of TFP for South Korea (left panel) and the US (right panel) under various assumptions. The solid line shows the evolution of TFP in the data, de-trended by the growth rate of TFP along a BGP, $\gamma/(\theta(1-\beta))$. The other lines correspond to simulations using alternative values of the diffusion parameters β. The case of $\beta = 0$ (dotted line) gives the dynamics of TFP implied by a standard Ricardian trade model, for example, the dynamics quantified by Connolly and Yi (2015). The other two lines illustrate the dynamic gains from trade implied by the model, that is, the contribution from trade to TFP growth as measured by equation (20).

Two clear messages stem from this figure. First, for a wide range of values of the diffusion parameter the dynamic model accounts for a substantial fraction of the TFP dynamics of South Korea. This is particularly true when considering intermediate values of the diffusion parameters β. Recall from Section 3 that for an economy that is moderately open, dynamic gains from trade are nonmonotonic in β. Second, the right panel shows that changes in the dynamic gains from trade identified by the model are less relevant for understanding the growth experience of a developed country, which started with a relatively large stock of knowledge, and thus had less to learn from others.

Figure 3 shows the evolution of TFP for a larger set of Asian countries that experienced high growth in the post-war period. For each country, the solid line is the data, while the dotted line is the model with $\beta = 0$ when trade costs are adjusted, but the arrival rates of ideas are held fixed at the 1962 values, that is, the static gains from trade. The dashed line shows the evolution of TFP for the simple calibration of the diffusion parameter,

[50]In Table 1 of the Online Appendix I.2, we present summary statistics for each country in our sample.

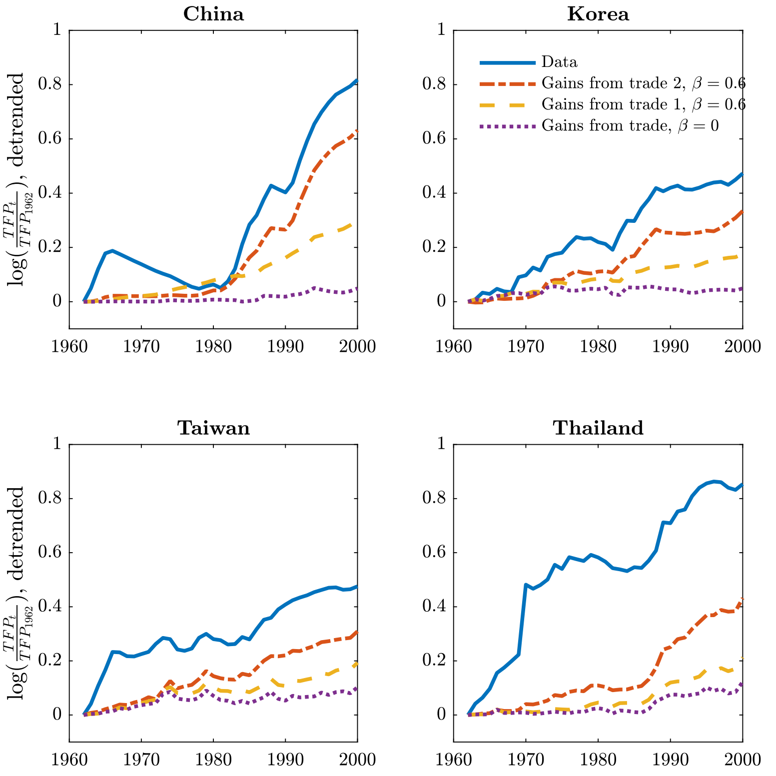

FIGURE 3.—Growth Miracles. **Note**: This figure plots the changes in detrended TFP for four miracle economies under the specification of learning from sellers. In each panel, we plot four lines. The solid line is the actual change in TFP. The dotted line is the contribution of trade in an environment where $\beta = 0$, that is, the static gains from trade. The other two lines show the results of two alternative counterfactuals incorporating dynamics gains under the assumption that $\beta = 0.6$. The dashed line shows the contribution from trade to the change in TFP as described by equation (20), that is, the change in TFP when only trade costs change. Finally, the dash-dotted line shows the contribution from trade to the change in TFP as described by equation (21), that is, when α changes. In all cases, TFP is detrended by the trend growth rate of TFP along a balanced growth path, $\gamma/(\theta(1-\beta))$.

$\beta = 0.6$. For some countries such as South Korea and China, the diffusion of ideas due to trade explains a substantial fraction of TFP growth. For others, such as Thailand changes in trade costs account for a smaller, but significant, fraction of TFP growth. Finally, the dash-dotted line shows the contribution of trade as measured by (21), that is, the evolution of TFP net of the contribution of changes in the arrival rates of ideas. The fact that this second measure of the contribution of trade tends to be larger suggests strong complementarities between changes in trade costs and in the arrival rates of ideas.

The experience of growth miracles is useful to illustrate that dynamic gains are particularly large when moving away from autarky. Figure 4 illustrates the importance of dynamic gains relative to static gains from trade for countries that are initially closer to autarky in 1962. The x-axis shows the own trade share of countries in 1962. The y-axis shows the dynamic multiplier, that is, the ratio of the gains from trade under the assumption that $\beta = 0.6$ to the gains from trade under the assumption that $\beta = 0$, as measured by equation (20). In this figure, we only include countries for which the increase in TFP when the

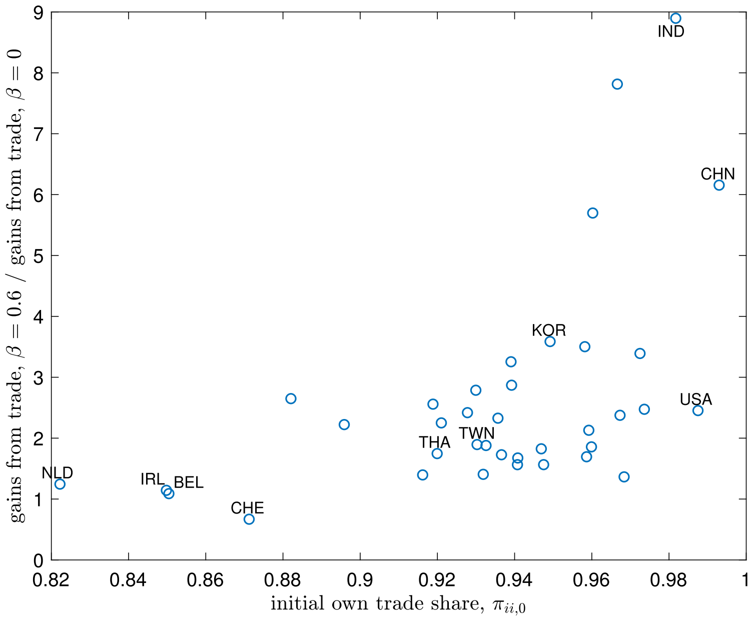

FIGURE 4.—Dynamic Multiplier and Initial Openness. **Note**: This figure illustrates the importance of dynamic gains relative to static gains from trade for countries that are initially closer to autarky in 1962. The x-axis shows the own trade of countries in 1962. The y-axis shows the dynamic multiplier, that is, the ratio of the contribution from trade under the assumption that $\beta = 0.6$ to the contribution from trade under the assumption that $\beta = 0$, as measured by equation (20), under the assumption that learning is from sellers and the arrival rate of ideas is kept at its 1962 value, $\alpha_{it} = \alpha_{i0}$. We only include countries for which the increase in TFP when the stocks of knowledge are held fixed are greater than 0.7%, the static gains experienced by India over the 1962–2000 period.

stocks of knowledge are held fixed are greater than 0.7%, the static gains experienced by India over the 1962–2000 period.

Among the initially most isolated countries, China and India stand out. These are also countries for which the total gains from trade are over 6 times larger than the static gains.[51] The US is another country that is initially very isolated, but it had also a relatively large stock of knowledge, so dynamic gains were not particularly prominent. Of the miracle economies highlighted in the quantitative section, Korea is initially among the more isolated economies, and it also featured large dynamic gains. Taiwan and, more prominently, Thailand were initially relatively more open economies and, therefore, had smaller dynamic gains. In the lower left side of the figure, are various economies that were very open in 1962 and, therefore, featured negligible dynamic gains from trade.[52]

4.4.2. *A Systematic Assessment*

We now perform a more systematic assessment of how the diffusion of ideas alters the explanatory power of trade in the model. We begin this analysis by comparing the static

[51] India was not among the miracle economies that we highlight in the quantitative section as it did not feature a large increase in TFP. Interestingly, India is not a country that opened its economy significantly during this period although, according to our model, it would have gained a lot from doing so.

[52] In the Online Appendix E.7, we also show that the prominence of dynamic gains from trade among growth miracles is associated with observed changes in their trade exposure toward more productive trading partners.

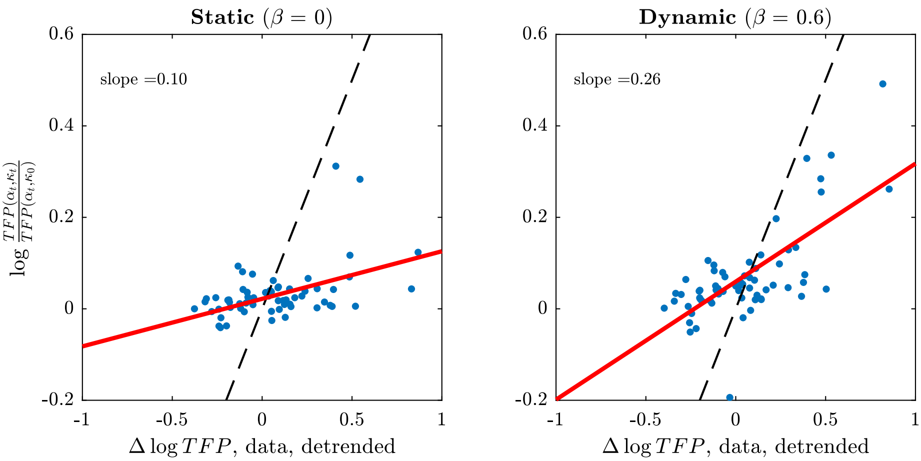

FIGURE 5.—Trade and the TFP Dynamics, 1962–2000. **Note**: Each panel plots countries' actual changes in TFP against the predicted changes in TFP in the model due to changes in trade costs. We compute this counterfactual under the assumptions that the arrival rates evolve as in our baseline calibration. In particular, the predicted changes in TFP due to changes in trade cost equals $\log[TFP(\alpha_t, \kappa_t)/TFP(\alpha_t, \kappa_0)]$, the second term in the right-hand side of equation (21). The first panel assumes that there are no dynamic gains from trade, $\beta = 0$. The second panel assumes $\beta = 0.6$. In addition, each figure plots a dashed 45-degree line and a red regression line.

and dynamic gains from changes in trade costs with observed changes in TFP for the calibrate value of $\beta = 0.6$. We then perform a more thorough analysis of the contribution of trade in accounting for world growth and the variance of TFP growth, for alternative values of the strength of diffusion β.

Figure 5 compares the predicted changes in TFP due to changes in trade costs, as measured by the second counterfactual in equation (21), to the changes in TFP observed in the data. We do this for a version of the model with no dynamic gains from trade, $\beta = 0$, and for the calibrated model, $\beta = 0.6$. Each point represents a country, and each panel contains a regression line through the observations and a dashed 45-degree line. If changes in trade costs fully account for each country's TFP growth, each dot would be on the (dashed) 45 degree line. The red regression line in each panel provides a simple measure of the average ability of the theory to account for cross-country differences in TFP growth. In particular, the regression coefficient corresponds to a measure of the fraction of the variance of TFP growth accounted for by trade, as we explain below. The left panel shows the predicted changes in TFP when $\beta = 0$ so that there are no dynamic gains from trade. The model predicts that changes in trade cost only led to small changes in TFP, consistent with small static gains from trade, accounting for 10% of the variance of TFP growth. In the right panel, β is set to 0.6, the value implied by the simple calibration discussed at the end of Section 4.2. In this panel, the regression line is more upward sloping, with a slope of 0.26, indicating that the contribution of trade to changes in TFP more than doubles.

Figure 6 shows a more thorough assessment of how the strength of diffusion alters the explanatory power of trade in the model. We can summarize the role of trade in a few different ways. We first compute the fraction of changes in TFP growth accounted for by contributions from trade and from contributions from changes in the arrival rates of

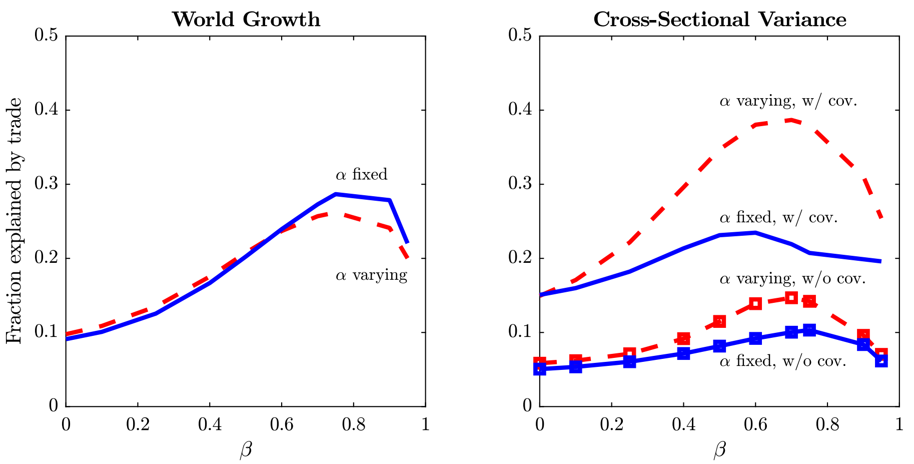

FIGURE 6.—The Contribution of Changes in Trade Costs to Changes in TFP. **Note**: This figure reports the fraction of TFP growth accounted for by trade costs, for various values of β, according to two decompositions. In both panels, the solid lines correspond to (20) in which the contribution of trade is evaluated holding the arrival rates constant; the dashed lines to correspond to (21) in which the contribution of trade is evaluated at the evolving arrival rates that are consistent with data. The left panel reports the fraction of total growth in TFP accounted for by changes in trade costs. The right panel reports the fraction of variance in TFP growth rates accounted for by changes in trade costs. The lines with square markers exclude the covariance between the contribution from trade and the contribution from changing arrival rates of ideas; the lines without markers include the covariance. In all cases, insights are drawn from sellers.

ideas.[53] We present these calculations in the left panel. The solid line corresponds to the first counterfactual in which the contribution of trade is evaluated at the initial arrival rates, and the dashed line corresponds to the second counterfactual in which contribution of trade is evaluated at the actual arrival rates.

According to this decomposition, both counterfactuals indicate that the static gains from trade ($\beta = 0$) account for roughly 8% of the growth in TFP from 1962–2000. With $\beta > 0$, changes in trade costs are more important. The contribution trade is highest if $\beta = 0.7$, a setting in which over a quarter of the increases in TFP are accounted for by changes in trade costs.

While the model predicts that changes in trade barriers can account for a significant fraction of TFP growth, it is possible that the model assigns growth to the wrong countries. To address this, the right panel of Figure 6 shows the fraction of variation in TFP growth rates accounted for trade costs. The variance of TFP growth can be decomposed into three components, the variance of contributions of changes in trade costs, the variance of the contributions of changes in arrival rates, and twice the covariance of the two. The figure plots four lines. The two solid lines correspond to the decomposition in (20) in which the contribution of trade is evaluated holding the arrival rates of ideas fixed at their initial levels. The two dashed lines correspond to (21) in which the contribution of trade is evaluated allowing the arrival rates to evolve as they must to explain the data. The lines that are marked with squares represent the fraction of variance of TFP growth rates

[53]For each of the two counterfactuals, this is $\frac{\sum_i \text{contribution from trade}}{\sum_i \ln \frac{TFP_i(\alpha_t,\kappa_t)}{TFP_i(\alpha_0,\kappa_0)}}$. It is thus a weighted average of the fraction of each country's TFP growth due to trade, where countries are weighted by their TFP growth.

accounted for by the variance of the contributions from trade. The lines without markers add in twice the covariance between the two contributions.[54]

Three lessons emerge. First, the contribution of trade to both the level and variation of TFP changes is greatest for intermediate values of the diffusion parameter, β. As highlighted in Section 3, for β close to 1 a country's stock of knowledge depends much more heavily on insights from the most productive producers, so that even countries close to autarky have accrued most of the dynamic gains from trade. Consequently, when β is close to 1, the model does not predict much dispersion in TFP growth among countries that are moderately open.

Second, the covariance terms are also quite large; countries whose TFP rose most saw increases stemming from trade but also from increasing the arrival of ideas. This is consistent with the notion that some countries reformed along many margins, which both increased trade and increased R&D. Including this covariance, changes in trade costs can account for more than a third of the variation of changes in TFP (when β is roughly 0.6). Alternatively, reducing trade barriers may raise incentives to innovate, at least along transition paths. A fuller exploration of this interaction merits further research.

Third, when the contribution of trade is evaluated at the arrival rates inferred from data, trade accounts for more of the variance of TFP changes (either including or excluding the covariance). This happens because changes in trade costs and in the arrival rates of ideas are complementary. Intuitively, improvements in the quality of insights matter more when the arrival rate of these insights is greater.

4.5. *Discussion of Alternative Specifications*

In this section, we focused on the case where insights are drawn from sellers to a market. As we have shown, this version of the model provides a promising quantitative theory of dynamic gains from trade, as it predicts large dynamic gains close to autarky and that the gains from trade depend on the composition of trading partners. Notwithstanding this, our analysis does not settle the question of which is the right specification of learning. It is therefore important to explore the implication of alternative specifications of learning.

In the Online Appendix E, we repeat the quantitative analysis for the case in which insights are drawn uniformly from producers. This is the model discussed in Section 2.2. In this version of the model, diffusion simply amplifies the static gains from trade. The recalibration of the implied path of the arrival rates are such that the contribution of trade increases only modestly with β. For all values of β, the contribution of trade in accounting for world growth is smaller in the model where insights are drawn uniformly from producers. The same is true of the contribution of trade in accounting for cross sectional distribution of TFP growth when considering intermediate values of the diffusion parameter β. These differences highlight the need for more research into the nature of learning.[55]

[54]The regression coefficient reported in Figure 5 is equal to an average of the two dash lines, that is, it equals the fraction of the variance of TFP growth rates accounted for by the variance of the contribution of trade plus the covariance between the contribution of trade and the contribution of changes in the arrival rates of ideas.

[55]In the Online Appendix E, we present results for two additional cases. We extend the learning from sellers specification to include targeted learning and study a case in which insights are drawn from domestic producers in proportion to labor used.

5. CONCLUSION

In this paper, we have provided a tractable theory of the diffusion of ideas across countries and a quantitative assessment of the role of trade in the transmission of knowledge. We found that when the model is specified so that the strength of diffusion is at an intermediate level, the model predicts a stronger response of TFP to changes in trade barriers than if the model were specified at either extreme of pure innovation or of pure diffusion. We showed quantitatively that the ability of trade barriers to account for changes in TFP from 1962–2000 is up to three times as large when the model allows for dynamics gains from trade. For our preferred calibration, we found that both gains from trade and the fraction of variation of TFP growth accounted for by changes in trade more than double relative to a model without diffusion.

The analysis points to critical importance of the strength of diffusion, β. While we provided one crude strategy to calibrate β, a more robust strategy would make better use of the variation in trade costs identified by Feyrer (2009a,b) or the evidence on changes in comparative advantage documented by Hanson, Lind, and Muendler (2015). The model also clarifies that there is no single effect of trade on growth; treatment effects are heterogeneous along many dimensions. The change in a country's income in response to lowering trade barriers depends on which other countries it trades with, which trading partners it opens trade with, how much it is trading with those countries already, and how far it is from its balanced growth path. A careful assessment of the relationship between trade and growth should recognize (and perhaps utilize) this heterogeneity.

Of course, we omitted many channels that may complement or offset the role of trade in the diffusion of ideas. Chief among these are FDI, purposeful imitation, or some other baseline level of interaction that is independent of trade. The productivity spillovers from trade are modeled as an external effect, which likely reflects how some but not all ideas diffuse. In addition, we have abstracted from variation across industries. Knowledge from one industry may be more useful in generating ideas to be used in the same industry than in other industries. The theory is tractable enough to incorporate many of these extensions. In light of this, our quantitative results assessing the role of openness should be viewed as a first step rather than the final word.

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